An Unusual Nine-Pointed Star Polygon Design of *La Alhambra*

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Abstract

This paper will explore a highly unusual, nine-pointed star polygon design found in two locations at *La Alhambra* - in a wooden ceiling of *El Mexuar* and also in a ceramic dado in the *Mirador de Lindaraja*. The design, which consists of both nine- and twelve-pointed regular star polygons, may be reconstructed using only the geometer's tools of straightedge and compass (or the electronic equivalent, the *Geometer's Sketchpad* software). Photographs of the ceiling and the ceramic dado will be shown and compared with idealized, computer-generated patterns, one colored and the other in skeletal form. Lastly, the design will be analyzed for geometric structure and crystallographic symmetry group classification..

Introduction

The highly symmetric, geometric designs of Islamic art may be recreated by constructing a geometrical grid, or a regular division of the plane, consisting of identical *repeat units*. These repeat units may be produced in a manner described by El-Said and Parman [1] using only a compass to draw circles and a straight edge to draw line segments. Once the design has been created by replication of the repeat unit, it may then be classified as belonging to one of the 17 possible crystallographic symmetry groups.

The author has reconstructed dozens of geometric Islamic designs ([2] – [7]), but until now, has not encountered a two-dimensional design consisting of a nine-pointed star polygon. This paper will illustrate two sources of the same, highly unusual star polygon design found at *La Alhambra*, in a wooden ceiling of *El Mexuar* and also in a ceramic tiling of the dado in the *Mirador de Lindaraja*. The design, which actually consists of both nine- and twelve-pointed regular star polygons, will be recreated using the *Geometer's Sketchpad* software program [8], analyzed for its symmetry elements, and then classified as to its crystallographic symmetry group.

El Mexuar's Wooden Ceiling

The nine- and twelve-pointed star polygon design, is found as part of a refurbished 16th century Moriscos-decorated, wooden ceiling panel in *El Mexuar* (see Figure 1. on the next page). The design consists of three different, but related, twelve-pointed star polygons embedded within each other, within a twelve-sided polygon (or "12-gon"). The smallest twelve-pointed star may be formed by offsetting four equilateral triangles by 30 degrees. This small star is embedded in a larger twelve-pointed star, which may be created by extending the edges of the smallest star until the lines intersect; these points of intersection form the vertices of the star. The third and largest twelve-pointed star is also produced by extending the edges of the second star, but to enclose the edges to form a "rose petal," extra intersecting lines in the shape of an "X" must be introduced at the point of tangency of a circle and a radius that intersects the vertices of the smallest star.

Surrounding each twelve-pointed star are six nine-pointed stars, the construction of which was not at all straightforward. In fact, those familiar with the famous problem of "Trisecting the Angle" may recall that the construction of a *regular* nine-sided polygon (ennagon, or "9-gon") is impossible using only a compass and straightedge. Nonetheless, the author was able to find a way to reconstruct a 9-gon that is consistent with the design and gives the appearance of congruent sides and angles; then within this 9-gon, the nine-pointed star design was recreated. An idealized, computer-generated rendition of this design, without the interlacing, is given in Figure 2 below.

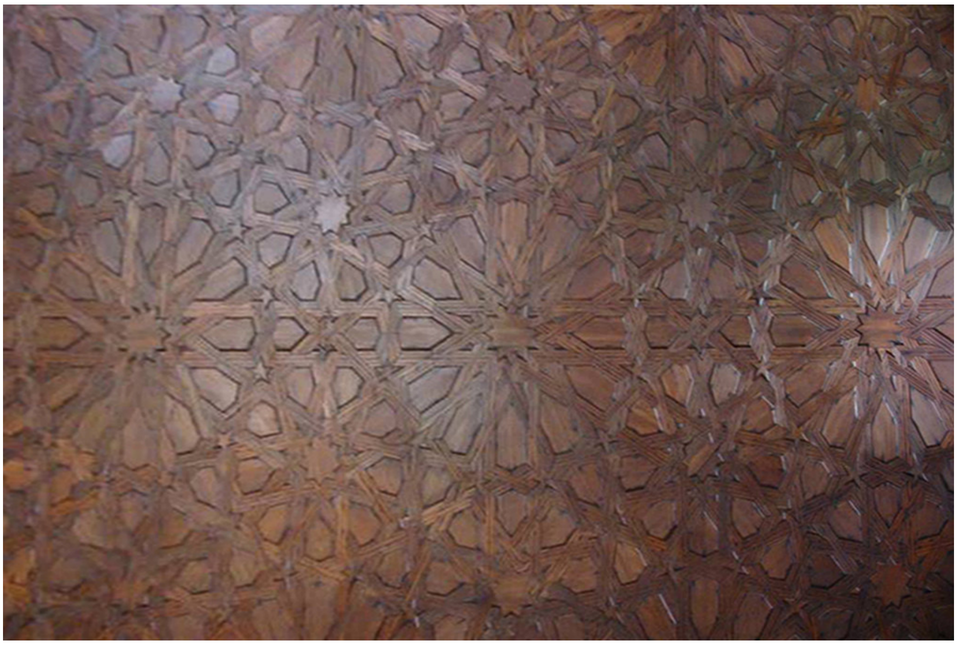

Figure 1. Wooden ceiling panel in *El Mexuar*

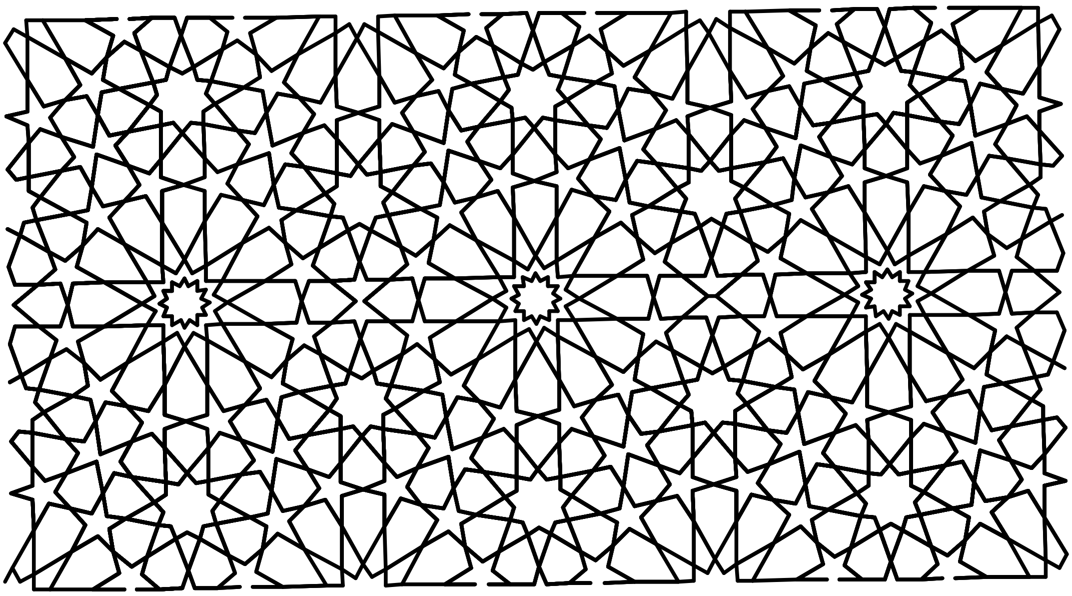

Figure 2. Idealized, computer-generated rendition of design in Figure 1., without the interlacing

If one focuses the center of Figure 2. above, the large twelve-pointed star is symmetrically surrounded by six nine-pointed stars (as illustrated in Figure 3. below) and therefore has six-fold symmetry. A less-cluttered view of the design, showing only the regular nine- and twelve-pointed stars is given in Figure 4. Since there are many possible mirror lines in this design (including the three shown in Figure 4.), the design may be classified as belonging to the of *p*6*m* crystallographic symmetry group.

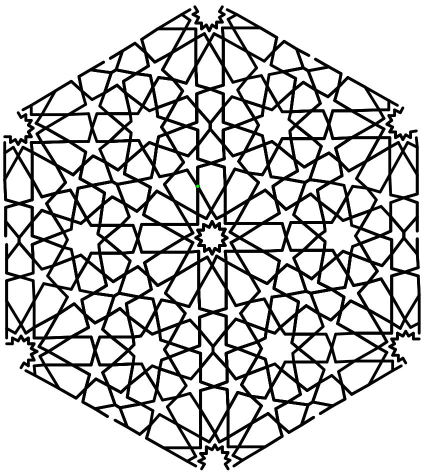

Figure 3.

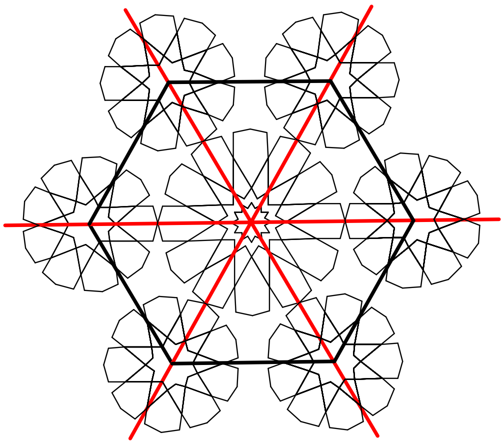

Figure 4.

Mirador de Lindaraja Ceramic Tiling

This nine- and twelve-pointed star polygon design is also found in a ceramic tiling on two adjacent and perpendicular walls in the corner of the *Mirador de Lindaraja* (see Figure 5. below). The designs on the two walls align perfectly to continue the overall pattern in an uninterrupted manner across the "crease" created by the corner. If the two wall segments were laid flat, the original design would look like the idealized, colored version of the one given in Figure 6. (Note, that the coloring would change the crystallographic symmetry group classification.)

Figure 5.

Figure 6.

Summary and Discussion

Most star polygon designs have an even number of points, although five-pointed stars (which are connected with the *Golden Section*) are also regularly found in geometric Islamic art. Nine-pointed stars designs, however, are quite rare in Islamic planar tilings. In fact, the only other nine-pointed star designs of which the author is aware are those that may be found on the 15th century curved surfaces of mausoleum domes in Cairo, Egypt and near Isfahan and in Mahan, Iran.

This twelve- and nine-pointed stars design may be classified as belonging to the *p*6*m* crystallographic symmetry group, and, as such, is one of the most common to Islamic cultures, where the preferred patterns are *p*6*m, p*4*m, cmm, pmm* and *p*6 types, with *p*4*m* and *p*6*m* predominating [9].

Recreating the entire design was extremely challenging. The construction of the nested twelve-pointed stars in a 12-gon was a relatively straight forward process, based upon two square grids inscribed within a circle. Two other nested stars were then created in a similar manner, with these three designs centered at the vertices of an equilateral triangle. A 9-gon was then formed within the triangle by connecting select, symmetrical vertices from the three 12-gons with line segments. Keeping in mind that the theoretical interior angle measure of a regular 9-gon is 140 degrees, the interior angle measure of six of the angles of the author's constructed 9-gon is 140.1 degrees, with the interior angle measure of the remaining three interior angles being 139.79 degrees. Six of the author's 9-gon's sides are congruent to each other, with the remaining three sides also being congruent to themselves. The measure of these two side lengths are within 2 percent of each other.

So, discovering two examples of the same nine-pointed star pattern at *La Alhambra*, one in a 16th century wooden ceiling and the other in a ceramic dado, presumably dating from the 15th century, was very intriguing and raises many questions worth investigating. What other nine-pointed star designs exist, from what location and time period? Did the Islamic artists construct their nine-pointed stars in the same manner as this author? And did they (or the mathematicians from whom they acquired their geometrical knowledge) know (or even care) that the 9-gon in this design was not regular?

References

[1] I. El-Said and A. Parman. *Geometric Concepts in Islamic Art.* Dale Seymour Publications, 1976.
[2] B. L. Bodner, *Star Polygon Designs of La Alhambra's Wooden Ceilings,* accepted for publication in the Bridges Conference Proceedings, 2004.
[3] B. L. Bodner, *The Geometrical Structure of Portal and Window Grilles of La Mezquita,* paper presented at the Joint Mathematical Association of America/American Mathematics Society meeting in Phoenix, Arizona, 2004.
[4] B. L. Bodner, *Constructing and Classifying Designs of al-Andalus*, Meeting Alhambra, ISAMA – Bridges Conference Proceedings, 2003.
[5] B. L. Bodner, *Mathematics of Islamic Art,* 45-minute invited talk and 1-hour invited workshop at the Mathematical Association of America New Jersey Section Conference, 2002.
[6] B. L. Bodner, *Islamic Art,* special 2-hour Special Invited Session at the Mathematical Association of America Mathfest Conference, 2001.
[7] B. L. Bodner, *Mathematics of Islamic Art,* paper presented at the Monmouth University Faculty Forum, 2001.
[8] The *Geometer's Sketchpad* (version 4) software, distributed by Key Curriculum Press, 2001.
[9] S. J. Abas and A. S. Salman, *Symmetries of Islamic Geometrical Patterns*. World Scientific, 1998.